# Combinatorics

## Semestral Examination

## B.Math. III

**Instructions:** All questions carry equal marks.

1. Prove that the existence of a $(t+1)-(v+1, k+1, \lambda)$ design implies the existence of a $t-(v, k, \lambda)$ design. Give an example to show that the converse is not true.

2. Define combinatorial geometry and geometric lattice. Prove that there is a one-one, onto correspondence between the set of finite combinatorial geometries and the set of finite geometric lattices.

3. Prove that the number of hyperplanes in a finite combinatorial geometry is at least the number of points of that geometry.

4. Define modular combinatorial geometry. If a modular geometry is a union of two flats, then prove that it is also a disjoint union of two flats.

5. Prove that any geometry in which the number of hyperplanes equals the number of points must be a modular geometry.

6. Let $F$ be a finite field. Prove that in the projective plane $PG_2(F)$, if two triangles are perspective from a point, then they are perspective from a line.